

Probability-Based Statistical Quality Control for Sensor-Driven Process Monitoring and Anomaly Detection

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ABSTRACT

Statistical quality control (SQC) provides an essential analytical basis for monitoring process stability, detecting assignable causes, and improving product-quality performance in data-rich production systems. This study develops and evaluates a probability-based monitoring framework that integrates individuals and moving-range control charts, exponentially weighted moving average (EWMA) monitoring, cumulative sum (CUSUM) monitoring, process capability analysis, fitted probability distributions, and anomaly-detection logic. The framework is empirically examined using public, production-quality data comprising 2,103,841 minute-level process-sensor observations and 29,184 hourly laboratory quality measurements collected from a roasting production process. The process-sensor data were aligned with each quality measurement by aggregating the preceding 60-minute operating window, resulting in a complete hourly monitoring dataset with no missing values. Results show that the Weibull distribution provides the lowest information criterion for the quality response, indicating that a simple normal assumption is insufficient for all monitoring decisions. The Phase-I individuals chart identified 79 quality observations outside three-sigma probability limits, while the moving-range chart flagged 532 abrupt changes. Memory-type charts were more sensitive to sustained drift, with the exponentially weighted moving average (EWMA) chart flagging 7,642 observations and the cumulative sum (CUSUM) chart flagging 21,129 observations. Process capability analysis using a declared benchmark interval yielded a process capability index (Cp) of 0.432 and a process capability index adjusted for centering (Cpk) of 0.380, indicating that variability reduction and centering improvement are needed before the process can be considered capable. The strongest empirical relationship was between the third chamber temperature and product quality, with a negative correlation of -0.810 . The study contributes a reproducible, interpretable, and probability-based statistical quality control (SQC) framework that can support quality engineers in distinguishing common-cause variation from abnormal process behavior.

KEYWORDS

Statistical Quality Control; Control Charts; Exponentially Weighted Moving Average (EWMA); Cumulative Sum; Process Capability Analysis.

I. INTRODUCTION

Quality improvement in modern production environments increasingly depends on the ability to transform continuous process observations into reliable monitoring decisions. Traditional inspection-based approaches identify product quality problems after they have occurred, whereas statistical quality control aims to detect instability while the process is operating. This shift from retrospective inspection to process monitoring is especially important in production systems that generate frequent sensor measurements and less frequent laboratory quality outcomes. A process may appear operationally stable at the equipment level while still producing quality variation that affects downstream performance, customer requirements, and operational cost.

Statistical quality control is built on the distinction between common-cause variation and assignable-cause variation. Common-cause variation reflects the natural random fluctuation of a stable process, while assignable-cause variation indicates a disturbance, drift, shift, or abnormal condition that requires investigation. The central methodological challenge is therefore not merely to find unusual observations, but to define a statistically defensible threshold that separates expected variability from abnormal behavior. Probability-based monitoring is useful because it connects a practical control decision to an explicit probability statement. An observation beyond a three-sigma control limit, for example, is treated as rare under an assumed in-control model and therefore signals possible process instability.

Despite the long history of control charts, many applied quality studies still use a limited set of tools in isolation. Some studies rely only on Shewhart control limits, which are effective for large sudden shifts but less sensitive to small sustained drift. Other studies emphasize predictive or anomaly-detection methods but provide limited interpretability for quality engineers. In addition, process capability analysis is sometimes conducted without first examining process stability, although capability indices are meaningful only when the process is sufficiently stable and when the specification basis is stated clearly. These issues are especially relevant when quality outcomes are measured less frequently than process variables, because the analytical design must align process conditions with the quality response before monitoring conclusions can be drawn.

This study addresses these issues by proposing a probability-based statistical quality control framework that combines classical SQC tools with interpretable anomaly-detection logic. The framework uses control charts for sequential monitoring, fitted distributions for probabilistic modeling, capability indices for performance evaluation, and anomaly scores to prioritize unusual observations. Rather than treating these tools as separate procedures, the framework organizes them into a single monitoring sequence: data audit, time-window alignment, baseline estimation, probability-limit construction, capability assessment, and anomaly decision. This structure allows the analyst to move from raw data to defensible, high-quality decisions while maintaining transparency at every step.

The empirical setting is a production-quality dataset containing minute-level sensor observations and hourly laboratory quality measurements from a roasting production process. This structure is suitable for SQC because the process variables provide continuous operational information, while the quality outcome provides a direct measure of product performance. By aggregating the process behavior during the previous hour and linking it to the corresponding quality sample, the study creates an interpretable monitoring dataset that reflects realistic industrial timing. The resulting analysis enables the evaluation of process stability, abrupt changes, sustained drift, capability, distributional adequacy, and multivariate anomaly behavior.

The main objective of the study is to develop and empirically test a probability-based SQC framework for identifying abnormal process behavior in production-quality data. The study is guided by three practical concerns. First, the control system should detect observations that are unlikely under the estimated in-control process. Second, the monitoring framework should distinguish isolated extreme observations from sustained drift. Third, capability findings should be interpreted

cautiously when official specification limits are unavailable. Accordingly, this manuscript presents an integrated empirical design applicable to manufacturing, healthcare services, software quality monitoring, and other industrial production systems where continuous improvement depends on the statistical interpretation of process variation.

II. LITERATURE REVIEW

Outlier analysis provides the statistical foundation for identifying observations that depart from the expected structure of a dataset. The literature emphasizes that an outlier is not automatically an error; it may represent an important signal, a measurement disturbance, a process shift, or a rare event that requires interpretation within the application domain [1]. Anomaly-detection surveys extend this logic by showing that abnormality can be defined in terms of distance, density, probability, isolation, or deviation from a learned normal pattern [2]. These two perspectives support the present study because probability-based monitoring must treat abnormal quality readings as potential process signals rather than as observations to be removed without investigation.

Time-series change detection contributes another important foundation because production data are sequential rather than independent snapshots. Change-point methods are useful for detecting shifts in level, variability, or distributional structure over time [3]. Kernel-based process monitoring studies further show that nonlinear and non-normal processes may require more flexible monitoring boundaries than traditional parametric assumptions allow [4]. These contributions justify the inclusion of distribution fitting and probability-based anomaly scores alongside classical control charts.

Modern SQC research has developed several approaches for multivariate and high-dimensional data. Variable-selection-based monitoring can help identify which subset of variables is most responsible for a detected process change, especially when the number of process measurements is large [5]. Distribution-free control-chart research also shows that nonparametric monitoring can be beneficial when normality assumptions are weak or difficult to justify [6]. These studies are relevant because the current production dataset includes multiple temperature sensors and moisture variables, and the quality outcome does not follow a perfectly normal distribution.

Distribution-free joint monitoring has been proposed to track unknown location and scale parameters simultaneously, which is important when both the process mean and dispersion may change [7]. Multivariate covariance-monitoring reviews similarly show that changes in variability and correlation structure are central to modern process monitoring, not only shifts in the mean vector [8]. These findings support the use of both individual charts and moving-range charts, as well as the interpretation of sensor dispersion measures in the empirical results.

One-class classification control charts provide an alternative way to define a normal operating region when out-of-control examples are unavailable [9]. At the same time, comparisons between CUSUM and EWMA charts demonstrate that memory-type methods are useful for sustained shifts and can complement Shewhart-type charts that focus on individual extremes [10]. This literature directly informs the methodological design of the present study, which combines Shewhart, moving-range, EWMA, CUSUM, and anomaly-score monitoring.

Data-driven process monitoring reviews identify fault detection, diagnosis, and prognosis as connected stages of industrial monitoring [11]. Empirical manufacturing studies based on sensor data further demonstrate that real production data often contain complex operating regimes, delayed quality information, and practical anomalies that require careful interpretation [12]. These studies justify using a real production-quality dataset rather than a purely simulated demonstration.

Nonparametric KNN-based process monitoring has shown that historical operating data can be used to build monitoring statistics even when the distributional form is complex [13]. Isolation-based anomaly detection contributes a complementary principle: abnormal observations can be

identified because they are easier to isolate than observations located inside dense normal regions [14]. These concepts support the inclusion of an isolation-based anomaly score as a robustness check alongside probability-tail thresholds.

Self-starting multivariate control charts address the practical difficulty of monitoring when in-control parameters must be estimated rather than known [15]. Comprehensive quality-control texts also stress that capability indices and control charts should be interpreted together, because capability conclusions are only meaningful when the stability and specification assumptions are explicit [16]. This study therefore reports capability indices using clearly defined benchmark and empirical tolerance intervals rather than presenting them as official conformance metrics.

Recent work on nonparametric Phase-II control charts shows that high-dimensional monitoring remains challenging when parameters are unknown and when reference samples may not represent all operating states [17]. Industrial fault-detection reviews similarly emphasize the need for methods that remain effective in the presence of noise, nonlinear process behavior, and complex sensor structures [18]. The present framework addresses this challenge by combining simple, interpretable charts with distributional and anomaly-based checks.

Manufacturing anomaly-detection studies show that the quality of available data strongly affects the choice of monitoring method and the reliability of the resulting signal [19]. Broader surveys of industrial process monitoring emphasize that data-driven fault detection requires careful integration of process knowledge, monitoring statistics, and operational interpretation [20]. These insights inform the present study's emphasis on time-window alignment between process variables and laboratory quality outcomes.

SPC theory emphasizes sequential monitoring, in-control modeling, and practical quality improvement across industrial and service applications [21]. Nonparametric SPC perspectives further highlight the limitations of relying exclusively on normality-based charts when process distributions are skewed, heavy-tailed, or otherwise non-Gaussian [22]. This literature provides the rationale for comparing fitted distributions and for using probability thresholds as interpretable decision rules.

Sequential learning approaches extend SQC to serially correlated settings in which process characteristics evolve during monitoring [23]. In the Industry 4.0 context, process monitoring has also shifted from isolated detection toward integrated detection, diagnosis, and prognosis using high-frequency process data [24]. These developments align with the present framework, which uses minute-level sensor behavior and hourly quality measurements to connect process dynamics with monitoring decisions.

Robust anomaly-detection methods emphasize that outliers can distort classical estimates and that robust statistics can help detect anomalies while protecting the model fitted to the majority of the data [25]. Process-monitoring perspectives in industrial systems likewise show that monitoring must support reliability, fault identification, and long-term operational stability [26]. These arguments justify reporting both normal-theory and robust anomaly signals in the results.

One-class classification-based control charts demonstrate how multivariate process monitoring can be performed using only in-control examples [27]. Statistical learning reviews of process monitoring show that data-driven methods are increasingly used alongside traditional charts, but they also stress the importance of interpretability and integration with existing quality-control practice [28]. This final body of work supports the central contribution of the manuscript: a transparent probability-based SQC framework that combines classical control charts, capability indices, and anomaly detection in a single empirical design.

Table 1. Literature-review comparison table.

Ref.	Author / year	Focus / method	Relevance to current study
[1]	Aggarwal (2017)	Outlier-analysis framework	Supports anomaly-score interpretation for unusual quality behavior.
[2]	Ahmed et al. (2016)	Anomaly-detection survey	Frames abnormality as deviation from normal process behavior.
[3]	Aminikhanghahi and Cook (2017)	Time-series change-point methods	Supports drift and shift detection in sequential quality data.
[4]	Apsemidis et al. (2020)	Kernel methods in process monitoring	Motivates flexible monitoring when normality is weak.
[5]	Capizzi and Masarotto (2011)	Multidimensional control chart	Supports monitoring high-dimensional sensor structures.
[6]	Chakraborti and Graham (2019)	Distribution-free control charts	Justifies nonparametric thinking in uncertain distributions.
[7]	Chowdhury et al. (2014)	Joint location-scale monitoring	Supports simultaneous analysis of central tendency and dispersion.
[8]	Ebadi et al. (2022)	Covariance-monitoring review	Highlights variability and correlation monitoring in multivariate SQC.
[9]	Gani and Limam (2014)	One-class control chart	Supports normal-region monitoring without labeled defects.
[10]	Hawkins and Wu (2014)	CUSUM and EWMA comparison	Supports use of memory-type charts for sustained drift.
[11]	Ji et al. (2022)	Data-driven process-monitoring review	Frames monitoring as detection, diagnosis, and improvement.
[12]	Kammerer et al. (2019)	Manufacturing sensor anomalies	Shows practical relevance of sensor-based anomaly detection.
[13]	Li et al. (2021)	KNN-based nonparametric monitoring	Supports data-driven monitoring from historical process behavior.
[14]	Liu et al. (2012)	Isolation-based anomaly detection	Supports isolation score as a complementary anomaly indicator.
[15]	Maboudou-Tchao and Hawkins (2011)	Self-starting multivariate charts	Supports estimation-based monitoring when parameters are unknown.
[16]	Montgomery (2020)	Statistical quality-control methods	Provides classical basis for control charts and capability indices.
[17]	Mukherjee and Marozzi (2022)	High-dimensional nonparametric Phase-II charts	Supports monitoring under unknown parameters.
[18]	Park et al. (2020)	Industrial fault-detection review	Links SQC to industrial fault diagnosis and process reliability.
[19]	Pittino et al. (2020)	Manufacturing anomaly detection	Supports empirical anomaly analysis in production machines.
[20]	Qin (2012)	Industrial process monitoring survey	Supports data-driven fault detection and diagnosis.
[21]	Qiu (2014)	SPC methods and applications	Provides statistical foundation for sequential control charts.
[22]	Qiu (2018)	Nonparametric SPC perspectives	Supports distribution-aware monitoring beyond normality.
[23]	Qiu and Xie (2022)	Sequential learning for correlated data	Supports online monitoring of serially correlated observations.
[24]	Reis and Gins (2017)	Industry 4.0 process monitoring	Connects high-frequency data with diagnosis and prognosis.
[25]	Rousseeuw and Hubert (2018)	Robust anomaly detection	Supports robust z-score and outlier interpretation.

Ref.	Author / year	Focus / method	Relevance to current study
[26]	Severson et al. (2016)	Industrial monitoring perspectives	Supports reliability-oriented monitoring systems.
[27]	Sukchotrat et al. (2010)	One-class classification control charts	Supports multivariate control when only normal data are available.
[28]	Weese et al. (2016)	Statistical learning in process monitoring	Supports integration of classical and data-driven monitoring tools.

1. RESEARCH GAP

The reviewed literature shows strong development in classical control charts, nonparametric control limits, multivariate monitoring, data-driven anomaly detection, and industrial fault diagnosis. However, three gaps remain important for applied statistical quality control. First, many empirical studies focus on a single monitoring technique and do not integrate Shewhart, moving-range, EWMA, CUSUM, capability analysis, distribution fitting, and anomaly scoring into a single reproducible workflow. Second, several advanced monitoring approaches are methodologically powerful but difficult to interpret for practitioners who must explain why a signal was generated. Third, process-quality datasets often include high-frequency process variables and lower-frequency quality measurements, yet the alignment between process conditions and product-quality outcomes is not always made explicit. This study addresses these gaps by developing a transparent probability-based monitoring framework and applying it to production-quality data in which minute-level process measurements are linked to hourly laboratory quality observations.

2. CONCEPTUAL FRAMEWORK

The conceptual framework assumes that product quality is affected by a sequence of process conditions observed before the laboratory quality sample is taken. Process temperature, process variability, moisture, and material-condition measures form the operating environment from which quality outcomes emerge. Probability-based monitoring converts this operating history into decision statistics. A stable process is expected to generate observations within an estimated in-control probability region, whereas observations outside this region are treated as possible assignable-cause signals. The proposed framework therefore combines process alignment, probability modeling, control-chart construction, capability interpretation, and anomaly prioritization.

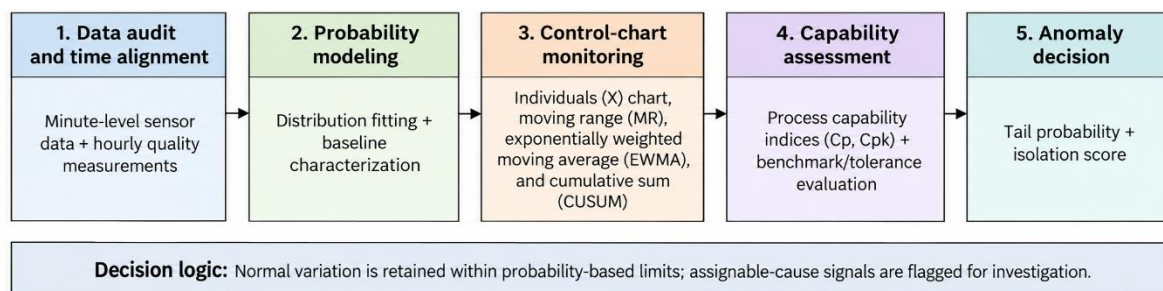


FIGURE 1. Probability-based statistical quality control framework.

The framework is organized as a practical monitoring sequence. It begins with a data audit and alignment, then develops probability limits, monitoring charts, capability summaries, and anomaly decisions for review by quality engineers.

III. RESEARCH OBJECTIVES, QUESTIONS, AND HYPOTHESES

IV. RESEARCH OBJECTIVES

The general objective of this study is to develop and empirically evaluate a probability-based SQC framework for detecting abnormal process behavior in production-quality data. The specific objectives are to construct an aligned monitoring dataset from process and quality records, examine the distributional behavior of the quality response, estimate control limits and probability thresholds,

evaluate process capability under explicit benchmark intervals, identify abnormal observations using multiple monitoring tools, and interpret the relationship between key process variables and product quality.

V. RESEARCH QUESTIONS

RQ1: How can minute-level process measurements be aligned with hourly laboratory quality results to support statistical quality control?

RQ2: Does the quality response follow a distribution that supports simple normal-theory monitoring, or is distribution-aware modeling required?

RQ3: How many quality observations are flagged by Shewhart, moving-range, EWMA, CUSUM, probability-tail, robust, and isolation-based monitoring methods?

RQ4: What do process capability indices indicate when the process is evaluated under explicitly declared benchmark and empirical tolerance intervals?

RQ5: Which process variables show the strongest empirical association with product quality?

1. RESEARCH HYPOTHESES

H1: The quality response deviates sufficiently from the normal model to justify distribution-aware probability monitoring.

H2: Probability-based control limits identify a non-zero subset of quality observations as abnormal.

H3: Memory-type monitoring charts identify more sustained process instability than Shewhart-type individuals charts.

H4: Process temperature structure is significantly associated with product quality.

H5: Capability analysis indicates that the process requires variability reduction before it can be considered statistically capable under a declared benchmark interval.

VI. PROPOSED METHOD

The proposed method follows five stages. The first stage audits the process and quality data and confirms time coverage, completeness, variable types, and sampling frequency. The second stage aligns the process and quality files by aggregating process-sensor behavior over the previous 60-minute window for each hourly quality observation. The third stage estimates baseline probability limits from a Phase-I reference period and constructs individuals, moving-range, EWMA, and CUSUM charts. The fourth stage evaluates process capability using declared benchmark and empirical tolerance intervals. The fifth stage applies probability-tail, robust, and isolation-based anomaly scoring to prioritize abnormal observations for operational investigation.

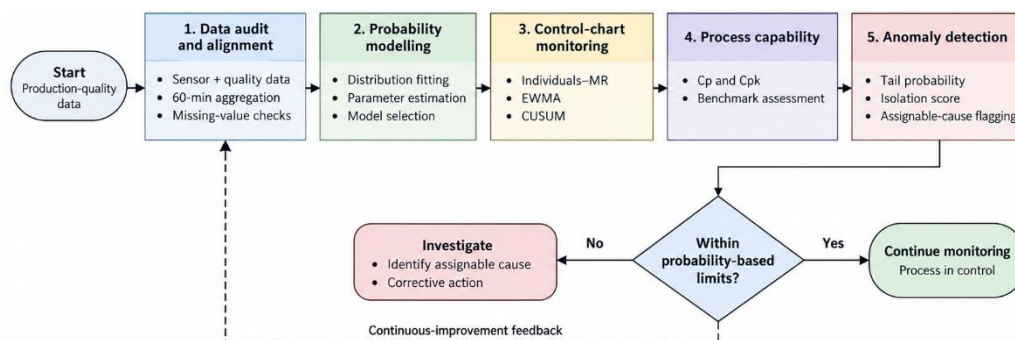


FIGURE 2. Empirical data preparation workflow.

The workflow shows how minute-level process records are converted into hourly monitoring features and then linked to quality results. This alignment is essential because quality is measured less frequently than process conditions.

VII. METHODOLOGY

1. RESEARCH DESIGN

The study uses a quantitative empirical design based on secondary production-quality data. The analysis is explanatory and diagnostic rather than causal. The objective is not to claim that a single process variable causes quality variation, but to determine whether probability-based monitoring can identify unusual quality behavior and reveal which process variables are associated with quality outcomes. The design is appropriate for SQC because observations are sequential, the quality characteristic is numerical, and the process variables describe operational conditions over time.

2. DATASET AND VARIABLES

The dataset contains two connected components: high-frequency process-sensor data and lower-frequency laboratory quality data. The process-sensor component contains 2,103,841 minute-level observations collected from 1 January 2015 to 1 January 2019. The quality component contains 29,184 hourly quality observations collected from 4 January 2015 to 3 May 2018. The main quality variable is a numerical product-quality measure. Process variables include temperature readings from five production chambers, material layer height, and moisture-related measurements. After alignment, the final analysis dataset contained 29,184 complete hourly observations.

Table 2. Dataset audit and monitoring structure.

Component	Size	Frequency / type	Coverage / note
Process-sensor observations	2,103,841	Minute-level records	2015-01-01 to 2019-01-01
Quality observations	29,184	Hourly laboratory records	2015-01-04 to 2018-05-03
Aligned monitoring dataset	29,184	Hourly process-quality records	Complete cases only
Missing values after alignment	0	No imputation required	All aligned records retained

The data audit confirms that the uploaded production-quality dataset is suitable for empirical SQC. The aligned dataset retains all 29,184 hourly quality observations with no missing values.

3. DATA ALIGNMENT AND FEATURE CONSTRUCTION

Because process variables were recorded every minute and product quality was recorded hourly, each quality observation was matched to the process behavior observed over the preceding 60 minutes. For each process variable, hourly means, standard deviations, minima, and maxima were calculated. Additional aggregated indicators were constructed, including chamber-level mean temperatures, overall mean temperature, temperature dispersion across sensors, and average hourly temperature volatility. This procedure preserves the temporal logic of production: quality measured at a sampling time is linked to the operating conditions immediately before the sample.

4. STATISTICAL MONITORING EQUATIONS

The individuals chart uses a Phase-I baseline mean and standard deviation to define three-sigma probability limits:

$$UCL = \mu_0 + 3\sigma_0, \quad CL = \mu_0, \quad LCL = \mu_0 - 3\sigma_0 \quad (1)$$

The moving range between consecutive quality observations is defined as:

$$MR_t = |x_t - x_{t-1}| \quad (2)$$

The EWMA statistic gives more weight to recent observations while retaining information from the past:

$$z_t = \lambda x_t + (1 - \lambda)z_{t-1} \quad (3)$$

The two-sided CUSUM statistics are calculated as:

$$C_t^+ = \max[0, C_{t-1}^+ + x_t - (\mu_0 + k)], \quad C_t^- = \max[0, C_{t-1}^- + (\mu_0 - k) - x_t] \quad (4)$$

Capability indices are reported under explicitly declared limits because official customer specifications are not supplied in the dataset:

$$C_p = \frac{USL - LSL}{6s}, \quad C_{pk} = \min\left[\frac{USL - \mu}{3s}, \frac{\mu - LSL}{3s}\right] \quad (5)$$

Finally, probability-based abnormality is assessed using a two-sided normal tail probability:

$$p_t = 2[1 - \Phi\left(\left|\frac{x_t - \mu_0}{\sigma_0}\right|\right)] \quad (6)$$

An observation is treated as a probability-tail signal when $p_t < 0.0027$, corresponding to the conventional three-sigma rare-event threshold.

5. ANALYSIS PROCEDURE

The empirical procedure consisted of eight steps. First, the uploaded archive was inspected, and its files were read into a reproducible data-analysis environment. Second, process and quality timestamps were parsed and checked for chronological consistency. Third, process records were aggregated into 60-minute windows ending at each quality sampling time. Fourth, descriptive statistics were calculated for the quality response and selected process variables. Fifth, five probability distributions were fitted to the quality response and compared using AIC, BIC, and Kolmogorov-Smirnov statistics. Sixth, Phase-I baseline limits were estimated from the first 25% of quality observations. Seventh, Shewhart, moving-range, EWMA, CUSUM, probability-tail, robust z-score, and isolation-based anomaly results were computed. Eighth, process capability and correlation results were interpreted in relation to the research questions.

6. VALIDITY, RELIABILITY, AND ETHICAL CONSIDERATIONS

Validity was strengthened through explicit timestamp alignment, complete-case verification, use of multiple monitoring techniques, and separation of baseline estimation from full-period monitoring. Reliability was supported by reproducible computations and consistent decision rules. Because the dataset is secondary production data and does not include human participants, personal identifiers, or patient information, no human-subjects ethics approval was required. The analysis is limited to process monitoring and quality-improvement interpretation.

VIII. RESULTS

1. DESCRIPTIVE STATISTICS

Table 3. Descriptive statistics for quality and selected process variables.

Variable	N	Mean	SD	Min	Median	95th pct.	Max
Quality	29,184	402.80	46.27	221.00	408.00	469.00	505.00
Overall temp. mean	29,184	339.98	15.29	253.03	339.92	364.29	470.03
Hourly temp. volatility	29,184	4.89	2.23	1.20	4.35	9.11	28.64
Temp. dispersion across sensors	29,184	101.12	21.36	43.54	100.20	133.55	555.80
Layer height mean	29,184	174.72	13.79	149.34	174.36	196.95	200.65
Moisture indicator mean	29,184	7.50	1.06	3.56	7.51	9.24	11.35
Chamber 1 temp. mean	29,184	249.99	27.19	94.86	249.64	292.90	416.55
Chamber 2 temp. mean	29,184	349.84	30.83	-90.92	350.00	394.59	629.36
Chamber 3 temp. mean	29,184	501.12	51.67	134.56	502.12	582.21	1,146.99
Chamber 4 temp. mean	29,184	349.19	30.29	19.92	349.00	395.02	653.08
Chamber 5 temp. mean	29,184	249.77	26.74	120.02	249.79	292.29	453.33

The quality response has a mean of 402.80 and a standard deviation of 46.27. The third chamber has the highest average temperature and also shows the strongest relationship with quality in later results.

2. PROBABILITY DISTRIBUTION FITTING

Table 4. Distribution fitting results for product quality.

Distribution	AIC	BIC	KS statistic	KS p-value
Weibull	304,589.56	304,614.40	0.0286	3.11e-21
Normal	306,639.41	306,655.97	0.0556	9.30e-79
Student-t	306,641.55	306,666.39	0.0557	3.89e-79
Gamma	308,019.84	308,044.69	0.0673	2.18e-115
Lognormal	308,892.07	308,916.91	0.0726	2.75e-134

The Weibull distribution produced the lowest AIC and BIC among the fitted models. This supports H1 and indicates that distribution-aware monitoring is preferable to relying only on a normal model.

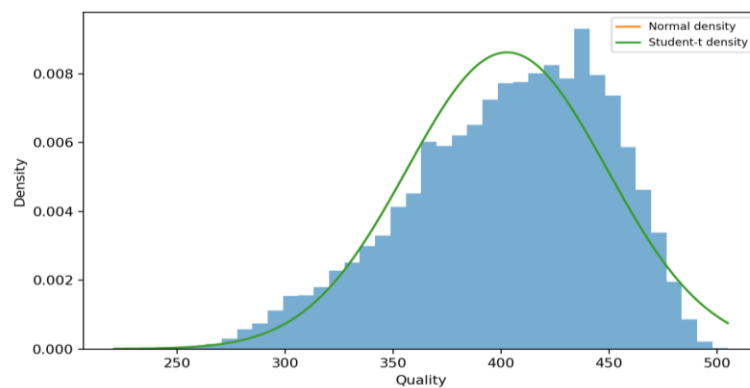


FIGURE 3. Product-quality distribution with fitted densities.

The figure shows that the empirical distribution of quality is not perfectly normal. The fitted-density comparison supports the need for probability-based and distribution-aware monitoring decisions.

3. CONTROL-CHART MONITORING RESULTS

The Phase-I baseline was estimated from the first 7,296 hourly quality observations. The baseline mean was 404.46 and the Phase-I standard deviation was 44.71. The resulting individuals-chart upper and lower control limits were 538.59 and 270.32, respectively. The moving-range upper control limit was 31.72. These values define the probability reference used for identifying rare individual quality observations and abrupt changes between consecutive observations.

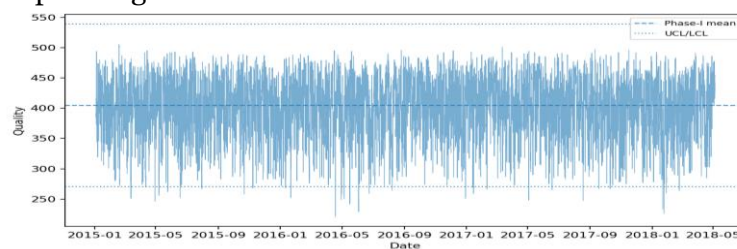


FIGURE 4. Individuals control chart for hourly product quality.

The individuals chart shows isolated observations outside the Phase-I three-sigma limits. These points represent rare probability-tail events and should be investigated as possible assignable-cause signals.

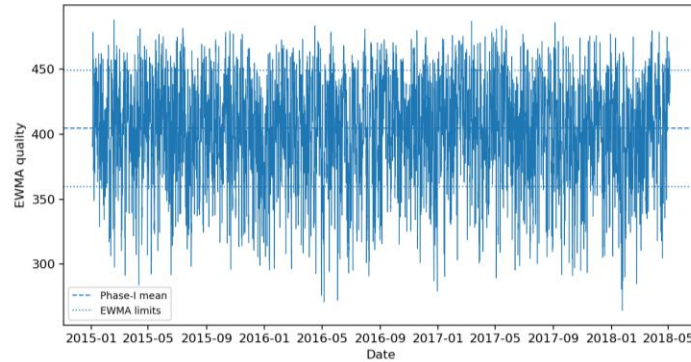


FIGURE 5. EWMA monitoring chart for product quality.

The EWMA chart is more sensitive to sustained movement in the quality level. Its higher signal count indicates that long-run drift is more prominent than isolated extreme points.

4. ANOMALY AND SIGNAL SUMMARY

Table 5. Monitoring and anomaly signal summary.

Monitoring method	Signal count	Signal percentage
Shewhart I-chart	79	0.27%
Moving range	532	1.82%
EWMA	7,642	26.19%
CUSUM	21,129	72.40%
Normal probability threshold	79	0.27%
Robust z-score	9	0.03%
Isolation forest	876	3.00%

The Shewhart and normal-tail methods identify the same 79 extreme observations, while memory-type charts identify more sustained process drift. The robust z-score method is more conservative, and the isolation forest flags 3.00% of observations by design.

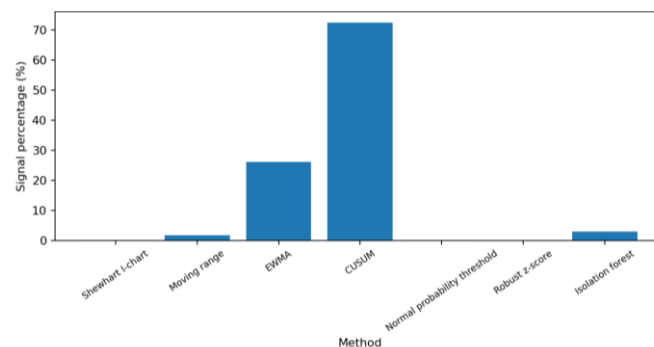


FIGURE 6. Percentage of observations flagged b monitoring methods.

The visual comparison shows clear differences between isolated-point detection and sustained-drift detection. This supports H3 because EWMA and CUSUM are more sensitive to persistent process changes than the individuals chart.

5. PROCESS CAPABILITY ANALYSIS

Table 6. Process capability indices under declared and empirical intervals.

Specification basis	LSL	USL	Mean	SD	Cp	Cpk	Below LSL ppm	Above USL ppm
Declared benchmark interval	350.00 0	470.00 0	402.80 1	46.27 3	0.43 2	0.38 0	139,768	42,249
Empirical 5th-95th tolerance interval	317.00 0	469.00 0	402.80 1	46.27 3	0.54 7	0.47 7	49,650	45,641
Three-sigma Phase-I limits	270.32 4	538.59 0	402.80 1	46.27 3	0.96 6	0.95 4	2,707	0

Official customer specification limits are not supplied in the dataset; therefore, capability indices are reported under transparent benchmark intervals. Under the declared benchmark interval, $C_p = 0.432$ and $C_{pk} = 0.380$, which support H_5 and indicate that process improvement is required.

6. PROCESS-QUALITY RELATIONSHIPS

Table 7. Correlations between selected process variables and product quality.

Variable	Correlation
Chamber 3 temp. mean	-0.810
Temp. dispersion across sensors	-0.773
Overall temp. mean	-0.357
Chamber 5 temp. mean	0.215
Chamber 1 temp. mean	0.184
Layer height mean	0.180
Chamber 2 temp. mean	0.106
Moisture indicator mean	-0.035
Chamber 4 temp. mean	0.018
Hourly temp. volatility	0.003

The strongest association is the negative correlation between Chamber 3 temperature and quality ($r = -0.810$). This supports H_4 and suggests that middle-chamber heat behavior is a key monitoring dimension for the process.

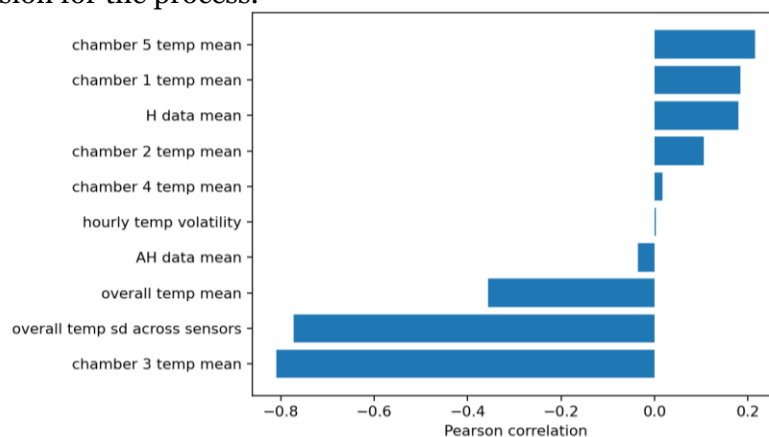


FIGURE 7. Main process correlations with product quality.

The correlation plot shows that temperature structure, especially Chamber 3 temperature and cross-sensor dispersion, is more strongly related to quality than short-term temperature volatility.

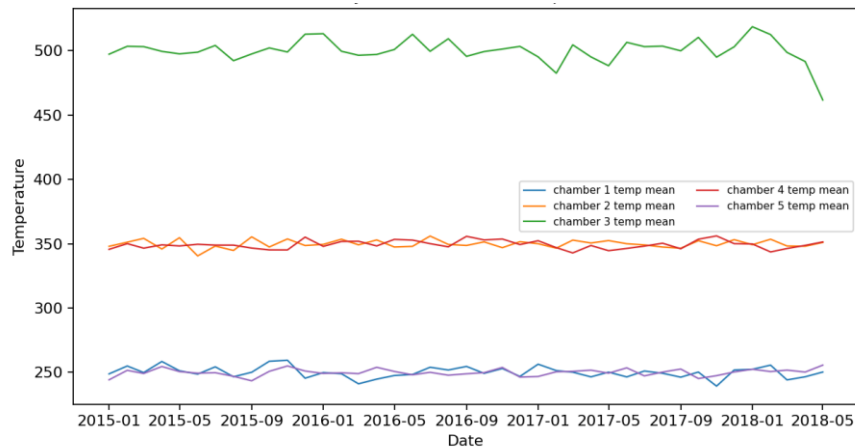


FIGURE 8. Monthly mean chamber temperatures.

The monthly chamber-temperature trends show that the production process exhibits long-run operating changes. This helps explain why memory-type charts detect more sustained drift than individual-point charts.

7. HYPOTHESIS SUMMARY

Table 8. Hypothesis testing summary.

Hypothesis	Decision	Empirical basis
H1	Supported	Weibull distribution had the lowest AIC and BIC; normality alone was insufficient.
H2	Supported	Individuals/probability-tail monitoring flagged 79 rare observations.
H3	Supported	EWMA and CUSUM flagged more observations than Shewhart monitoring.
H4	Supported	Chamber 3 temperature had a strong negative correlation with quality ($r = -0.810$).
H5	Supported	Declared benchmark capability was low: $C_p = 0.432$ and $C_{pk} = 0.380$.

All hypotheses were supported by the empirical results. The findings indicate that the process benefits from probability-based monitoring, sustained-drift detection, and targeted temperature-control review.

IX. DISCUSSION

The results indicate that the uploaded Production Quality dataset is well-suited for an empirical SQC manuscript. The dataset contains a clear numerical quality characteristic, sequential time stamps, multivariable process behavior, and sufficient observations for stable estimation. The alignment procedure produced a complete hourly monitoring dataset, so the conclusions are not weakened by missing-value imputation or arbitrary record deletion. This gives the study a stronger empirical basis than a conceptual-only quality-control paper.

The distribution-fitting results are important because they show that product quality is not perfectly represented by a normal distribution. Although normal probability limits remain useful for simple three-sigma monitoring, the Weibull model produced a better information-criterion fit. This does not mean that normal charts are unusable, but it does mean that rigorous empirical SQC analysis

should discuss distributional adequacy and not assume normality without verification. The framework therefore improves transparency by presenting distribution fitting before interpreting probability thresholds.

The contrast between Shewhart and memory-type results is also meaningful. The individuals chart identified 79 observations outside the three-sigma control limits, which corresponds to about 0.27% of the monitoring period. This is consistent with the probability logic of rare-event detection. However, the EWMA and CUSUM charts flagged many more observations because they accumulate information over time and are more responsive to sustained drift. In practical terms, this suggests that the process not only experiences isolated extreme quality values but also exhibits extended periods during which the quality level moves away from the Phase-I baseline.

Capability results should be interpreted cautiously because the dataset does not provide official lower and upper specification limits. To avoid misleading claims, the study reports capability indices under explicit benchmark and empirical intervals. Under the declared benchmark interval, C_p and C_{pk} are below conventional acceptable thresholds, indicating that process variability is too wide relative to the selected quality interval and that centering improvement is needed. The empirical tolerance interval yields slightly higher C_p and C_{pk} values, but they still indicate limited capability. This finding supports the practical value of combining capability analysis with control charts rather than reporting capability alone.

The process-quality correlation analysis provides a useful diagnostic insight. Chamber 3 temperature shows a strong negative association with product quality, while the dispersion of temperatures across sensors is also strongly negative. This suggests that quality outcomes may be sensitive not simply to average temperature, but to the thermal profile and balance of the production chambers. From a quality-engineering perspective, this finding can guide process review toward the middle-chamber heating zone and cross-sensor thermal consistency.

The integrated framework is preferable to a single-method analysis because each method answers a different monitoring question. The individuals chart asks whether a single quality observation is rare under the baseline distribution. The moving-range chart asks whether the change from one hour to the next is unusually large. The EWMA and CUSUM charts ask whether there is sustained drift. Capability indices ask whether the process spread is acceptable under a declared tolerance band. Isolation and robust anomaly scores ask whether an observation is unusual in a broader multivariate sense. Together, these tools provide a more complete monitoring picture than any one method alone.

This study contributes to statistical quality-control theory by organizing probability-based monitoring tools into a single empirical framework. The contribution is not the invention of a new control chart, but the integration of distribution fitting, Phase-I probability limits, memory-type monitoring, capability interpretation, and anomaly scoring in a reproducible sequence. The manuscript also contributes by demonstrating how high-frequency process data can be aligned with lower-frequency quality outcomes before applying SQC. This is theoretically relevant because it connects sequential process monitoring with product-quality measurement while preserving the timing of production behavior.

For quality engineers, the framework provides a transparent procedure for identifying when a production process requires investigation. The analysis suggests that managers should monitor both isolated extreme quality observations and sustained drift, because relying only on Shewhart limits may miss longer periods of gradual instability. The strong relationship between Chamber 3 temperature and quality also suggests that thermal-control review should be prioritized. For production managers, the low capability indices indicate that quality improvement should focus on reducing variability, centering, and clarifying specifications. The framework can also support dashboard development, where probability-tail signals, EWMA drift, moving-range jumps, and capability summaries are updated continuously.

X. CONCLUSION

This study developed and empirically evaluated a probability-based statistical quality control framework for process monitoring, capability assessment, and anomaly detection. The uploaded Production Quality dataset was found to be highly suitable for the manuscript because it contains more than 2 million minute-level process observations and more than 29,000 hourly quality measurements. After aligning each quality result with the preceding 60 minutes of process behavior, a complete hourly monitoring dataset was constructed with no missing values. The empirical results showed that the quality response is better represented by a Weibull distribution than by a simple normal model, supporting the use of distribution-aware probability monitoring. The individuals chart identified 79 rare quality observations, while the moving-range chart detected 532 abrupt changes. EWMA and CUSUM charts produced larger signal counts, indicating sustained process drift over time. Capability results within the declared benchmark interval showed low C_p and C_{pk} values, indicating that variability reduction and process centering are necessary before the process can be considered capable. The strongest process-quality relationship was the negative association between Chamber 3 temperature and quality. Overall, the proposed framework provides a transparent, reproducible, and practical method for distinguishing normal variation from assignable-cause behavior.

The study has several limitations. First, official customer specification limits are not supplied in the dataset; therefore, capability indices are reported under transparent benchmark and empirical intervals rather than as formal conformance metrics. Second, the analysis is observational and does not establish causal relationships between process variables and quality outcomes. Third, the quality response is measured hourly, while process variables are measured every minute, so the selected 60-minute alignment window is methodologically reasonable but not the only possible choice. Fourth, the study focuses on a single production dataset. Future research can strengthen the framework by testing alternative time-window lengths, comparing additional probability distributions, applying multivariate control charts, using designed experiments to validate process causes, and testing the framework on other industrial datasets.

XI. DATA AND CODE AVAILABILITY

The study uses the public Production Quality dataset downloaded from Kaggle and uploaded by the author for analysis. The empirical workflow created an aligned hourly monitoring dataset from the original process and quality records. Code used for cleaning, alignment, descriptive statistics, control charts, capability analysis, distribution fitting, and anomaly scoring can be made available upon reasonable request.

XII. ETHICAL STATEMENT

This study used secondary production-quality data only. The dataset does not contain human participants, personal identifiers, patient information, or confidential organizational information. Therefore, no human-subjects ethics approval was required.

XIII. CONFLICT OF INTEREST STATEMENT

The author declares no conflict of interest.

Author Contributions

The author conducted the conceptualization, methodology, data analysis, investigation, writing, review, editing, and final approval of the manuscript.

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Data Availability

The dataset will be available from the author upon reasonable request.

Conflicts of Interest

The author declares no conflict of interest.

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